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КОМПЬЮТЕРНОЕ МОДЕЛИРОВАНИЕ ДИНАМИКИ ПОЛУПРОВОДНИКОВОЙ ПЛАЗМЫ В ДВУМЕРНОМ СЛУЧАЕ

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Постановка задачи:

$$
\begin{cases}
\dfrac{\partial^2\varphi}{\partial x^2}+\dfrac{\partial^2\varphi}{\partial y^2}=\gamma\,(n-N) \\
\dfrac{\partial n}{\partial t}=D_x\dfrac{\partial}{\partial x}\left(\dfrac{\partial n}{\partial x}-\mu n\dfrac{\partial\varphi}{\partial x}\right)+D_y\dfrac{\partial}{\partial y}\left(\dfrac{\partial n}{\partial y}-\mu n\dfrac{\partial\varphi}{\partial y}\right)+G\,(N,n,\varphi)-R\,(n,N) \\
\dfrac{\partial N}{\partial t}=G\,(N,n,\varphi)-R\,(n,N) \\
\dfrac{\partial I}{\partial y}+\delta_0\delta(N,n,\varphi)I=0
\end{cases}
\qquad
\begin{matrix} 0<x<L_x \\ 0<y<L_y \\ t>0 \end{matrix}
$$

Краевые и начальные условия:

$$\left.\frac{\partial\varphi}{\partial x}\right|_{x=0,L_x}=-E_0 \qquad \left.D_x\left(\frac{\partial n}{\partial x}-\mu n\frac{\partial\varphi}{\partial x}\right)\right|_{x=0,L_x}=0 \qquad 0<y<L_y$$

$$\left.\frac{\partial\varphi}{\partial y}\right|_{y=0,L_y}=0 \qquad \left.D_y\left(\frac{\partial n}{\partial y}-\mu n\frac{\partial\varphi}{\partial y}\right)\right|_{y=0.L_y}=0 \qquad 0<y<L_y \qquad R=\frac{nN-n_0^2}{\tau_p}$$

$$I\big|_{y=0}=\exp\left(-\left(\frac{x-L_x/2}{0.1L_x}\right)^2\right)(1-\exp(-10t)) \qquad 0<x<L_x \qquad G=q_0 I\delta(N,n,\varphi)$$

$$\delta\,(n,N,\varphi)=(1-N)\exp(-\psi(1-\xi n))$$

$$
\begin{aligned}
&n\big|_{t=0}=N\big|_{t=0}=n_0\,(x,y)=n_0e^{\mu\varphi} \\
&\varphi\big|_{t=0}=-E_0x \\
&I\big|_{t=0}=0
\end{aligned}
\qquad
\begin{matrix} 0<x<L_x \\ 0<y<L_y \end{matrix}
$$

Построение разностной схемы:

На области $G = [0 \le x \le L_x] \times [0 \le y \le L_y] \times [0 \le t \le L_t]$

введем равномерные сетки $\Omega = \omega_x \times \omega_y \times \omega_t,\ \Omega' = \omega_x \times \omega_y \times \omega'_t,\ \Omega'' = \omega_x \times \omega'_y \times \omega'_t$:

$$\omega_x = \{x_i = ih_x, i = \overline{0, N_x}, h_x = L_x / N_x\},$$
$$\omega_y = \{y_j = jh_y, j = \overline{0, N_y}, h_y = L_y / N_y\},$$
$$\omega'_y = \{y'_j = (j-0.5)h_y, j = \overline{0, N_y+1}, h_y = L_y / N_y\},$$
$$\omega_t = \{t_k = k\tau, k = \overline{0, N_t}, \tau = L_t / N_t\},$$
$$\omega'_t = \{t'_k = (k+0.5)\tau, k = \overline{0, N_t - 1}, \tau = L_t / N_t\}.$$

Определим сеточные функции, на Ω: $n_{ij}^k = n(x_i, y_j, t_k),\ N_{ij}^k = N(x_i, y_j, t_k),\ \varphi_{ij}^k = \varphi(x_i, y_j, t_k)$.

на Ω': $\overline{n}_{ij}^k = n(x_i, y_j, t'_k)$.

на Ω'': $I_{ij}^k = I(x_i, y'_j, t'_k)$.

Обозначения:

$f = f_{ij}^k,\ f_{i\pm1} = f_{i\pm1 j}^k,\ f_{j\pm1} = f_{ij\pm1}^k,$

$f_{j\pm0.5} = 0.5\left(f_{ij}^k + f_{ij\pm1}^k\right),\ f_{i\pm0.5} = 0.5\left(f_{ij}^k + f_{i\pm1 j}^k\right),$

$\hat{f} = f_{ij}^{k+1},\ \overset{0.5}{f} = 0.5\left(f + \hat{f}\right),\ f = n, N, \varphi.$

$I = I_{ij}^k,\ I_{i\pm1} = I_{i\pm1 j}^k,\ \hat{I} = I_{ij+1}^k,\ \overset{0.5}{I} = 0.5\left(I + \hat{I}\right),$

$R = (N - N_0)^2/\tau_p,\ \hat{R} = (\hat{N} - N_0)^2/\tau_p,\ \overset{0.5}{R} = 0.5(R + \hat{R}),$

$G = q_0 \overset{0.5}{I}\delta,\quad \hat{G} = q_0 \overset{0.5}{I}\hat{\delta},\quad \overset{0.5}{G} = 0.5(G + \hat{G}),$

$\delta = (1 - N)e^{-\psi(1-\xi n)},\quad \delta = (1 - \hat{N})e^{-\psi(1-\xi\hat{n})},\quad \overset{0.5}{\delta} = 0.5(\delta + \hat{\delta}).$

Инвариант: $Q(t) = \sum_{i=1}^{N_x - 1}\sum_{j=1}^{N_y - 1} h_x h_y (n_{ij} - N_{ij})$.

Схема 1:

$$\frac{\hat{N}-N}{\tau}=\overset{0.5}{G}-\overset{0.5}{R}$$

$$\frac{\bar{n}-n}{0.5\tau}=D_x\bar{n}_{\bar{x}x}-D_x\mu\left(n_{i+0.5}\frac{\varphi_{i+1}-\varphi_i}{h_x^2}-n_{i-0.5}\frac{\varphi_i-\varphi_{i-1}}{h_x^2}\right)+D_y n_{\bar{y}y}-D_y\mu\left(n_{j+0.5}\frac{\varphi_{j+1}-\varphi_j}{h_y^2}-n_{j-0.5}\frac{\varphi_j-\varphi_{j-1}}{h_y^2}\right)+G-R$$

$$\frac{\hat{n}-\bar{n}}{0.5\tau}=D_x\bar{n}_{\bar{x}x}-D_x\mu\left(\hat{n}_{i+0.5}\frac{\hat{\varphi}_{i+1}-\hat{\varphi}_i}{h_x^2}-\hat{n}_{i-0.5}\frac{\hat{\varphi}_i-\hat{\varphi}_{i-1}}{h_x^2}\right)+D_y\hat{n}_{\bar{y}y}-D_y\mu\left(\hat{n}_{j+0.5}\frac{\hat{\varphi}_{j+1}-\hat{\varphi}_j}{h_y^2}-\hat{n}_{j-0.5}\frac{\hat{\varphi}_j-\hat{\varphi}_{j-1}}{h_y^2}\right)+\hat{G}-\hat{R}$$

$$\hat{\varphi}_{\bar{x}x}+\hat{\varphi}_{\bar{y}y}=\gamma\left(\hat{n}-\hat{N}\right)$$

$$\frac{\hat{I}-I}{h_y}+\delta_0\overset{0.5}{\delta}\overset{0.5}{I}=0$$

Краевые условия:

$$\frac{\hat{\varphi}_{1j}-\hat{\varphi}_{0j}}{h_x}=\frac{\hat{\varphi}_{N_xj}-\hat{\varphi}_{N_x-1j}}{h_x}=-E_0 \qquad j=1,\dots,N_y-1$$

$$\frac{\hat{\varphi}_{i1}-\hat{\varphi}_{i0}}{h_y}=\frac{\hat{\varphi}_{iN_y}-\hat{\varphi}_{iN_y-1}}{h_y}=0 \qquad i=1,\dots,N_x-1$$

$$\mu\frac{\left(n_{0.5j}+\hat{n}_{0.5j}\right)}{2}E_0+\frac{\bar{n}_{1j}-\bar{n}_{0j}}{h_x}=0 \qquad j=1,\dots,N_y-1$$

$$\mu\frac{\left(n_{N_x-0.5j}+\hat{n}_{N_x-0.5j}\right)}{2}E_0+\frac{\bar{n}_{N_xj}-\bar{n}_{N_x-1j}}{h_x}=0 \qquad j=1,\dots,N_y-1$$

$$\frac{\hat{n}_{i1}-\hat{n}_{i0}}{h_y}=\frac{\hat{n}_{iN_y}-\hat{n}_{iN_y-1}}{h_y}=0 \qquad i=1,\dots,N_x-1$$

$$I_{i0}^k=\exp\left(-\left(\frac{i-N_x/2}{0.1N_x}\right)^2\right)\left(1-\exp\left(-10k\tau\right)\right) \qquad i=1,\dots,N_x-1$$

Схема 1 с итерациями:

$$\frac{\overset{s+1}{\hat{N}}_i - N_i}{\tau} = \overset{s}{\overset{0.5}{G}} - \overset{s}{\overset{0.5}{R}}$$

$$\frac{\bar{n} - n}{0.5\tau} = D_x \bar{n}_{\bar{x}x} - D_x \mu \left(n_{i+0.5} \frac{\varphi_{i+1} - \varphi_i}{h_x^2} - n_{i-0.5} \frac{\varphi_i - \varphi_{i-1}}{h_x^2} \right) + D_y n_{\bar{y}y} - D_y \mu \left(n_{j+0.5} \frac{\varphi_{j+1} - \varphi_j}{h_y^2} - n_{j-0.5} \frac{\varphi_j - \varphi_{j-1}}{h_y^2} \right) + G - R$$

$$\frac{\overset{s+1}{\hat{n}} - \bar{n}}{0.5\tau} = D_x \bar{n}_{\bar{x}x} - D_x \mu \left(\overset{s}{\hat{n}}_{i+0.5} \frac{\overset{s}{\hat{\varphi}}_{i+1} - \overset{s}{\hat{\varphi}}_i}{h_x^2} - \overset{s}{\hat{n}}_{i-0.5} \frac{\overset{s}{\hat{\varphi}}_i - \overset{s}{\hat{\varphi}}_{i-1}}{h_x^2} \right) + D_y \overset{s+1}{\hat{n}}_{\bar{y}y} - D_y \mu \left(\overset{s}{\hat{n}}_{j+0.5} \frac{\overset{s}{\hat{\varphi}}_{j+1} - \overset{s}{\hat{\varphi}}_j}{h_y^2} - \overset{s}{\hat{n}}_{j-0.5} \frac{\overset{s}{\hat{\varphi}}_j - \overset{s}{\hat{\varphi}}_{j-1}}{h_y^2} \right) + \overset{s}{\hat{G}} - \overset{s}{\hat{R}}$$

$$\overset{s+1}{\hat{\varphi}}_{\bar{x}x} + \overset{s+1}{\hat{\varphi}}_{\bar{y}y} = \gamma \left(\overset{s+1}{\hat{n}} - \overset{s+1}{\hat{N}} \right)$$

$$\frac{\overset{s+1}{\hat{I}} - I}{h_y} + \delta_0 \overset{s+1}{\overset{0.5}{\delta}} \overset{s+1}{\overset{0.5}{I}} = 0$$

Краевые условия:

$$\frac{\overset{s+1}{\hat{\varphi}}_{1j} - \overset{s+1}{\hat{\varphi}}_{0j}}{h_x} = \frac{\overset{s+1}{\hat{\varphi}}_{N_x j} - \overset{s+1}{\hat{\varphi}}_{N_x-1 j}}{h_x} = -E_0 \qquad j = 1,\ldots, N_y - 1$$

$$\frac{\overset{s+1}{\hat{\varphi}}_{i1} - \overset{s+1}{\hat{\varphi}}_{i0}}{h_y} = \frac{\overset{s+1}{\hat{\varphi}}_{iN_y} - \overset{s+1}{\hat{\varphi}}_{iN_y-1}}{h_y} = 0 \qquad i = 1,\ldots, N_x - 1$$

$$\frac{\mu\left(n_{0.5j} + \overset{s}{\hat{n}}_{0.5j}\right)}{2} E_0 + \frac{\bar{n}_{1j} - \bar{n}_{0j}}{h_x} = 0 \qquad j = 1,\ldots, N_y - 1$$

$$\frac{\mu\left(n_{N_x-0.5j} + \overset{s}{\hat{n}}_{N_x-0.5j}\right)}{2} E_0 + \frac{\bar{n}_{N_x j} - \bar{n}_{N_x-1j}}{h_x} = 0 \qquad j = 1,\ldots, N_y - 1$$

$$\frac{\overset{s+1}{\hat{n}}_{i1} - \overset{s+1}{\hat{n}}_{i0}}{h_y} = \frac{\overset{s+1}{\hat{n}}_{iN_y} - \overset{s+1}{\hat{n}}_{iN_y-1}}{h_y} = 0 \qquad i = 1,\ldots, N_x - 1$$

$$I_{i0}^k = \exp\left(-\left(\frac{i - N_x/2}{0.1 N_x} \right)^2 \right) \left(1 - \exp\left(-10k\tau\right)\right) \qquad i = 1,\ldots, N_x - 1$$

Критерий окончания итерационного процесса:

$$\left| \overset{s+1}{\hat{n}} - \overset{s}{\hat{n}} \right| \le \varepsilon_1 \left| \overset{s}{\hat{n}} \right| + \varepsilon_2$$

$$\left| \overset{s+1}{\hat{N}} - \overset{s}{\hat{N}} \right| \le \varepsilon_1 \left| \overset{s}{\hat{N}} \right| + \varepsilon_2$$

$$\left| \overset{s+1}{\hat{\varphi}} - \overset{s}{\hat{\varphi}} \right| \le \varepsilon_1 \left| \overset{s}{\hat{\varphi}} \right| + \varepsilon_2$$

$$\left| \overset{s+1}{\hat{I}} - \overset{s}{\hat{I}} \right| \le \varepsilon_1 \left| \overset{s}{\hat{I}} \right| + \varepsilon_2$$

$$\varepsilon_1, \varepsilon_2 > 0.$$

Схема 2:

$$\frac{\hat{N}-N}{\tau}=\overset{0.5}{G}-\overset{0.5}{R}$$

$$\frac{\hat{n}-n}{\tau}=\frac{D_x}{2}\hat{n}_{\bar{x}x}+\frac{D_y}{2}\hat{n}_{\bar{y}y}+\frac{D_x}{2}n_{\bar{x}x}+\frac{D_y}{2}n_{\bar{y}y}-\frac{D_x\mu}{2}\left(n_{i+0.5}\frac{\varphi_{i+1}-\varphi_i}{h_x^2}-n_{i-0.5}\frac{\varphi_i-\varphi_{i-1}}{h_x^2}\right)-\frac{D_x\mu}{2}\left(\hat{n}_{i+0.5}\frac{\hat{\varphi}_{i+1}-\hat{\varphi}_i}{h_x^2}-\hat{n}_{i-0.5}\frac{\hat{\varphi}_i-\hat{\varphi}_{i-1}}{h_x^2}\right)-$$

$$-\frac{D_y\mu}{2}\left(n_{j+0.5}\frac{\varphi_{j+1}-\varphi_j}{h_y^2}-n_{j-0.5}\frac{\varphi_j-\varphi_{j-1}}{h_y^2}\right)-\frac{D_y\mu}{2}\left(\hat{n}_{j+0.5}\frac{\hat{\varphi}_{j+1}-\hat{\varphi}_j}{h_y^2}-\hat{n}_{j-0.5}\frac{\hat{\varphi}_j-\hat{\varphi}_{j-1}}{h_y^2}\right)+\overset{0.5}{G}-\overset{0.5}{R}$$

$$\hat{\varphi}_{\bar{x}x}+\hat{\varphi}_{\bar{y}y}=\gamma\left(\hat{n}-\hat{N}\right)$$

$$\frac{\hat{I}-I}{h_y}+\delta_0\overset{0.5}{\delta}\overset{0.5}{I}=0$$

Краевые условия:

$$\frac{\hat{\varphi}_{1j}-\hat{\varphi}_{0j}}{h_x}=\frac{\hat{\varphi}_{N_xj}-\hat{\varphi}_{N_x-1j}}{h_x}=-E_0 \qquad j=1,\dots,N_y-1$$

$$\frac{\hat{\varphi}_{i1}-\hat{\varphi}_{i0}}{h_y}=\frac{\hat{\varphi}_{iN_y}-\hat{\varphi}_{iN_y-1}}{h_y}=0 \qquad i=1,\dots,N_x-1$$

$$\mu\hat{n}_{0.5j}E_0+\frac{\hat{n}_{1j}-\hat{n}_{0j}}{h_x}=0 \qquad j=1,\dots,N_y-1$$

$$\mu\hat{n}_{N_x-0.5j}E_0+\frac{\hat{n}_{N_xj}-\hat{n}_{N_x-1j}}{h_x}=0 \qquad j=1,\dots,N_y-1$$

$$\frac{\hat{n}_{i1}-\hat{n}_{i0}}{h_y}=\frac{\hat{n}_{iN_y}-\hat{n}_{iN_y-1}}{h_y}=0 \qquad i=1,\dots,N_x-1$$

$$I_{i0}^k=\exp\left(-\left(\frac{i-N_x/2}{0.1N_x}\right)^2\right)\left(1-\exp\left(-10k\tau\right)\right) \qquad i=1,\dots,N_x-1$$

Схема 2, 1 этап:

$$\frac{\overset{s+1}{\hat{N}_i} - N_i}{\tau} = \overset{\overset{s}{0.5}}{G} - \overset{\overset{s}{0.5}}{R}$$

$$\frac{\overset{s+1}{\hat{n}} - n}{\tau} = \frac{D_x}{2}\overset{s+1}{\hat{n}}_{\bar{x}x} + \frac{D_y}{2}\overset{s}{\hat{n}}_{\bar{y}y} + \frac{D_x}{2} n_{\bar{x}x} + \frac{D_y}{2} n_{\bar{y}y} - \frac{D_x \mu}{2}\left(n_{i+0.5}\frac{\varphi_{i+1} - \varphi_i}{h_x^2} - n_{i-0.5}\frac{\varphi_i - \varphi_{i-1}}{h_x^2} \right) - \frac{D_x \mu}{2}\left(\overset{s}{\hat{n}}_{i+0.5}\frac{\overset{s}{\hat{\varphi}}_{i+1} - \overset{s}{\hat{\varphi}}_i}{h_x^2} - \overset{s}{\hat{n}}_{i-0.5}\frac{\overset{s}{\hat{\varphi}}_i - \overset{s}{\hat{\varphi}}_{i-1}}{h_x^2} \right) -$$

$$- \frac{D_y \mu}{2}\left(n_{j+0.5}\frac{\varphi_{j+1} - \varphi_j}{h_y^2} - n_{j-0.5}\frac{\varphi_j - \varphi_{j-1}}{h_y^2} \right) - \frac{D_y \mu}{2}\left(\overset{s}{\hat{n}}_{j+0.5}\frac{\overset{s}{\hat{\varphi}}_{j+1} - \overset{s}{\hat{\varphi}}_j}{h_y^2} - \overset{s}{\hat{n}}_{j-0.5}\frac{\overset{s}{\hat{\varphi}}_j - \overset{s}{\hat{\varphi}}_{j-1}}{h_y^2} \right) + \overset{\overset{s}{0.5}}{G} - \overset{\overset{s}{0.5}}{R}$$

$$\overset{s+1}{\hat{\varphi}}_{\bar{x}x} + \overset{s+1}{\hat{\varphi}}_{\bar{y}y} = \gamma\left(\overset{s+1}{\hat{n}} - \overset{s+1}{\hat{N}} \right)$$

$$\frac{\overset{s+1}{\hat{I}} - I}{h_y} + \delta_0 \overset{\overset{s+1}{0.5}}{\delta}\ \overset{\overset{s+1}{0.5}}{I} = 0$$

Краевые условия:

$$\frac{\overset{s+1}{\hat{\varphi}}_{1j} - \overset{s+1}{\hat{\varphi}}_{0j}}{h_x} = \frac{\overset{s+1}{\hat{\varphi}}_{N_x j} - \overset{s+1}{\hat{\varphi}}_{N_x - 1 j}}{h_x} = -E_0 \qquad j = 1,\dots, N_y - 1$$

$$\frac{\overset{s+1}{\hat{\varphi}}_{i1} - \overset{s+1}{\hat{\varphi}}_{i0}}{h_y} = \frac{\overset{s+1}{\hat{\varphi}}_{iN_y} - \overset{s+1}{\hat{\varphi}}_{iN_y - 1}}{h_y} = 0 \qquad i = 1,\dots, N_x - 1$$

$$\mu \overset{s}{\hat{n}}_{0.5j} E_0 + \frac{\overset{s+1}{\hat{n}}_{1j} - \overset{s+1}{\hat{n}}_{0j}}{h_x} = 0 \qquad j = 1,\dots, N_y - 1$$

$$\mu \overset{s}{\hat{n}}_{N_x - 0.5j} E_0 + \frac{\overset{s+1}{\hat{n}}_{N_x j} - \overset{s+1}{\hat{n}}_{N_x - 1j}}{h_x} = 0 \qquad j = 1,\dots, N_y - 1$$

$$I_{i0}^k = \exp\left(-\left(\frac{i - N_x/2}{0.1 N_x} \right)^2 \right)\left(1 - \exp\left(-10k\tau\right)\right) \qquad i = 1,\dots, N_x - 1$$

Схема 2, 2 этап:

$$\frac{\overset{s+2}{\hat{N}}_i - N_i}{\tau} = \overset{s+1}{\overset{0.5}{G}} - \overset{s+1}{\overset{0.5}{R}}$$

$$\frac{\overset{s+2}{\hat{n}} - n}{\tau} = \frac{D_x}{2}\overset{s+1}{\hat{n}}_{\bar{x}x} + \frac{D_y}{2}\overset{s+2}{\hat{n}}_{\bar{y}y} + \frac{D_x}{2} n_{\bar{x}x} + \frac{D_y}{2} n_{\bar{y}y} - \frac{D_x\mu}{2}\left(n_{i+0.5}\frac{\varphi_{i+1}-\varphi_i}{h_x^2} - n_{i-0.5}\frac{\varphi_i - \varphi_{i-1}}{h_x^2}\right) - \frac{D_x\mu}{2}\left(\overset{s+1}{\hat{n}}_{i+0.5}\frac{\overset{s+1}{\hat{\varphi}}_{i+1} - \overset{s+1}{\hat{\varphi}}_i}{h_x^2} - \overset{s+1}{\hat{n}}_{i-0.5}\frac{\overset{s+1}{\hat{\varphi}}_i - \overset{s+1}{\hat{\varphi}}_{i-1}}{h_x^2}\right) -$$

$$- \frac{D_y\mu}{2}\left(n_{j+0.5}\frac{\varphi_{j+1}-\varphi_j}{h_y^2} - n_{j-0.5}\frac{\varphi_j - \varphi_{j-1}}{h_y^2}\right) - \frac{D_y\mu}{2}\left(\overset{s+1}{\hat{n}}_{j+0.5}\frac{\overset{s+1}{\hat{\varphi}}_{j+1} - \overset{s+1}{\hat{\varphi}}_j}{h_y^2} - \overset{s+1}{\hat{n}}_{j-0.5}\frac{\overset{s+1}{\hat{\varphi}}_j - \overset{s+1}{\hat{\varphi}}_{j-1}}{h_y^2}\right) + \overset{s+1}{\overset{0.5}{G}} - \overset{s+1}{\overset{0.5}{R}}$$

$$\overset{s+2}{\hat{\varphi}}_{\bar{x}x} + \overset{s+2}{\hat{\varphi}}_{\bar{y}y} = \gamma\left(\overset{s+2}{\hat{n}} - \overset{s+2}{\hat{N}}\right)$$

$$\frac{\overset{s+2}{\hat{I}} - I}{h_y} + \delta_0 \overset{s+2}{\overset{0.5}{\delta}}\ \overset{s+2}{\overset{0.5}{I}} = 0$$

Критерий окончания итерационного процесса:

$$\left|\overset{s+2}{\hat{n}} - \overset{s}{\hat{n}}\right| \le \varepsilon_1\left|\overset{s}{\hat{n}}\right| + \varepsilon_2$$

$$\left|\overset{s+2}{\hat{N}} - \overset{s}{\hat{N}}\right| \le \varepsilon_1\left|\overset{s}{\hat{N}}\right| + \varepsilon_2$$

$$\left|\overset{s+2}{\hat{\varphi}} - \overset{s}{\hat{\varphi}}\right| \le \varepsilon_1\left|\overset{s}{\hat{\varphi}}\right| + \varepsilon_2$$

$$\left|\overset{s+2}{\hat{I}} - \overset{s}{\hat{I}}\right| \le \varepsilon_1\left|\overset{s}{\hat{I}}\right| + \varepsilon_2$$

$$\varepsilon_1, \varepsilon_2 > 0.$$

Краевые условия:

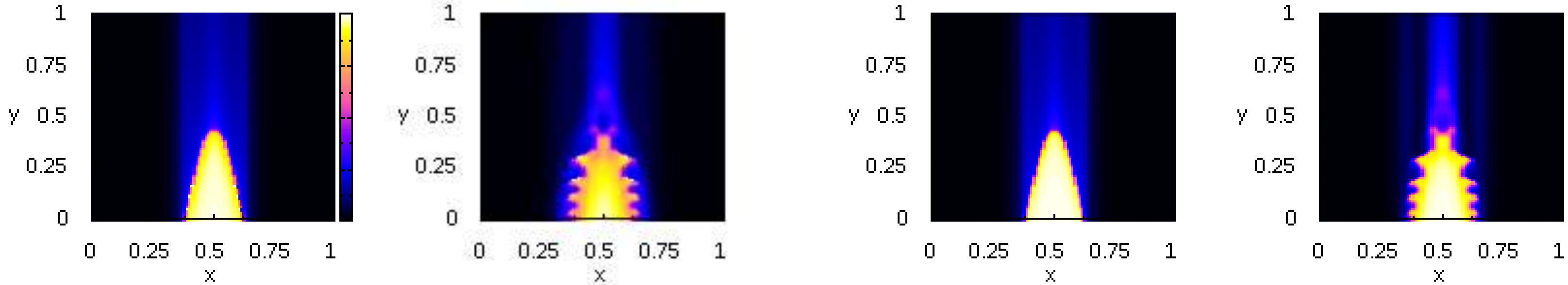
$$\frac{\overset{s+2}{\hat{\varphi}}_{1j} - \overset{s+2}{\hat{\varphi}}_{0j}}{h_x} = \frac{\overset{s+2}{\hat{\varphi}}_{N_x j} - \overset{s+2}{\hat{\varphi}}_{N_x-1 j}}{h_x} = -E_0 \qquad j = 1,\dots, N_y - 1$$

$$\frac{\overset{s+2}{\hat{\varphi}}_{i1} - \overset{s+2}{\hat{\varphi}}_{i0}}{h_y} = \frac{\overset{s+2}{\hat{\varphi}}_{iN_y} - \overset{s+2}{\hat{\varphi}}_{iN_y-1}}{h_y} = 0 \qquad i = 1,\dots, N_x - 1$$

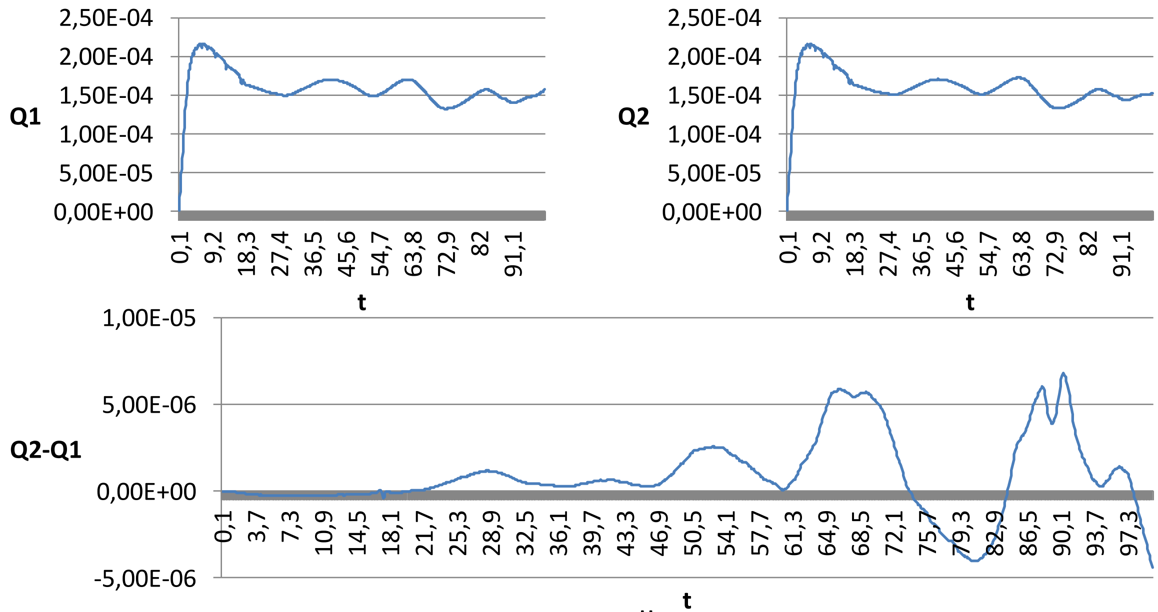
$$\frac{\overset{s+2}{\hat{n}}_{i1} - \overset{s+2}{\hat{n}}_{i0}}{h_y} = \frac{\overset{s+2}{\hat{n}}_{iN_y} - \overset{s+2}{\hat{n}}_{iN_y-1}}{h_y} = 0 \qquad i = 1,\dots, N_x - 1$$

$$I_{i0}^k = \exp\left(-\left(\frac{i - N_x/2}{0.1N_x}\right)^2\right)\left(1 - \exp(-10k\tau)\right) \qquad i = 1,\dots, N_x - 1$$

Сравнение точности сохранения инварианта:

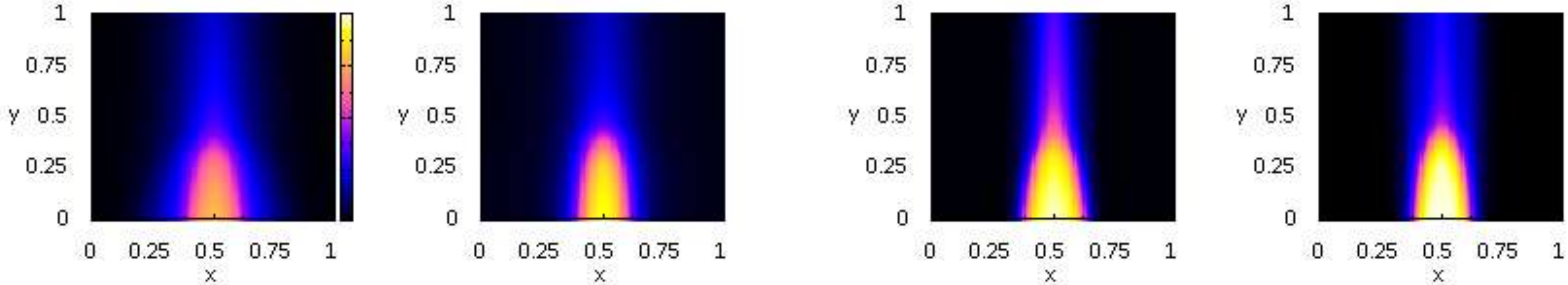


Распределения концентрации свободных электронов(a, b) и ионизированных доноров(c, d), реализуемые при значениях параметров $\delta_0 = 2,\ D_x = 10^{-5},\ D_y = 10^{-5},\ \gamma = 10^3,\ n_0 = 0.01,\ E_0 = 0,\ \mu = 1,$ $\psi = 2.553,\ \xi = 3,\ q_0 = 1.5,$ в моменты времени $t = 10$ (a, c), $t = 100$ (b, d).



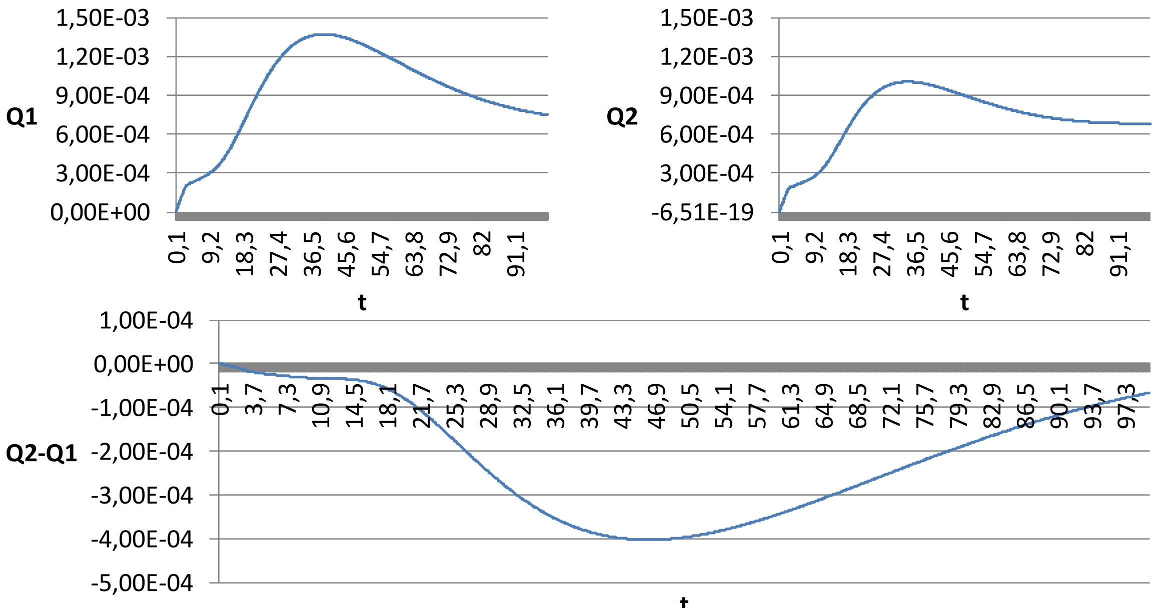
Распределения модулей инварианта $Q(t)$

Сравнение точности сохранения инварианта:



a) b) c) d)

Распределения концентрации свободных электронов(a, b) и ионизированных доноров(c, d), реализуемые при значениях параметров $\delta_0 = 2,\ D_x = 10^{-3},\ D_y = 10^{-5},\ \gamma = 10^3,\ n_0 = 0.01,\ E_0 = 0,\ \mu = 1,$ $\psi = 2.553,\ \xi = 3,\ q_0 = 1.5,$ в моменты времени $t = 10$ (a, c), $t = 100$ (b, d).



Распределения модулей инварианта $Q(t)$

Основные результаты:

- Разработаны 2 консервативные разностные схемы для двумерной задачи взаимодействия лазерного импульса с полупроводником.
- Построенные схемы реализованы в виде программы на языке C++.
- С помощью вычислительных экспериментов проведено сравнение эффективности схем.
- Получены различные режимы изменения характеристик полупроводника.

Публикации:

- В.А. Егоренков. *Разностная схема для расчета 2D генерации полупроводниковой плазмы, индуцированной лазерным импульсом.* // Сборник статей молодых ученых факультета ВМК МГУ, №10, 2013.

- V.A. Trofimov, M.M. Loginova, V.A. Egorenkov. *Conservative finite-difference schemes for 2D problem of femtosecond pulse propagation in semiconductor.* // Proceedings of the 13th International Conference on Mathematical Methods in Science and Engineering - Almeria, Spain.

- V.A. Trofimov, M.M. Loginova, V.A. Egorenkov. *Laser-induced 2D periodic structures of charged particles concentration in semiconductor under the condition of optical bistability existence.* // Abstracts of the International Conference “SPIE Optics + Photonics”. August 25-29, 2013. San Diego. USA. Book of abstracts on CD. Paper 8847-16.